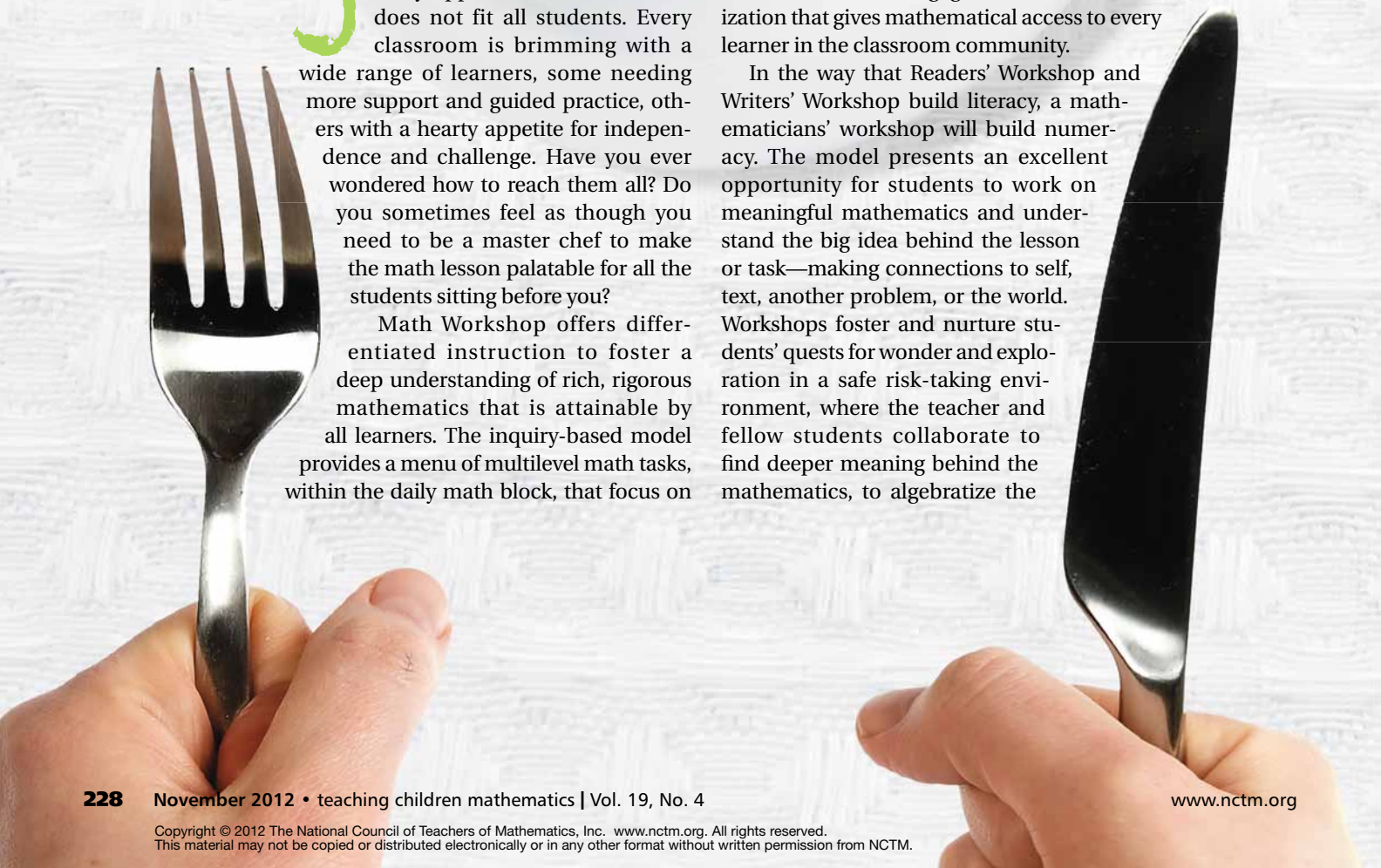
A first-grade teacher demonstrates how to serve up this model of inquiry-based instruction in any classroom.

Just as one meal does not satisfy every appetite, one math lesson does not fit all students. Every classroom is brimming with a wide range of learners, some needing more support and guided practice, others with a hearty appetite for independence and challenge. Have you ever wondered how to reach them all? Do you sometimes feel as though you need to be a master chef to make the math lesson palatable for all the students sitting before you?

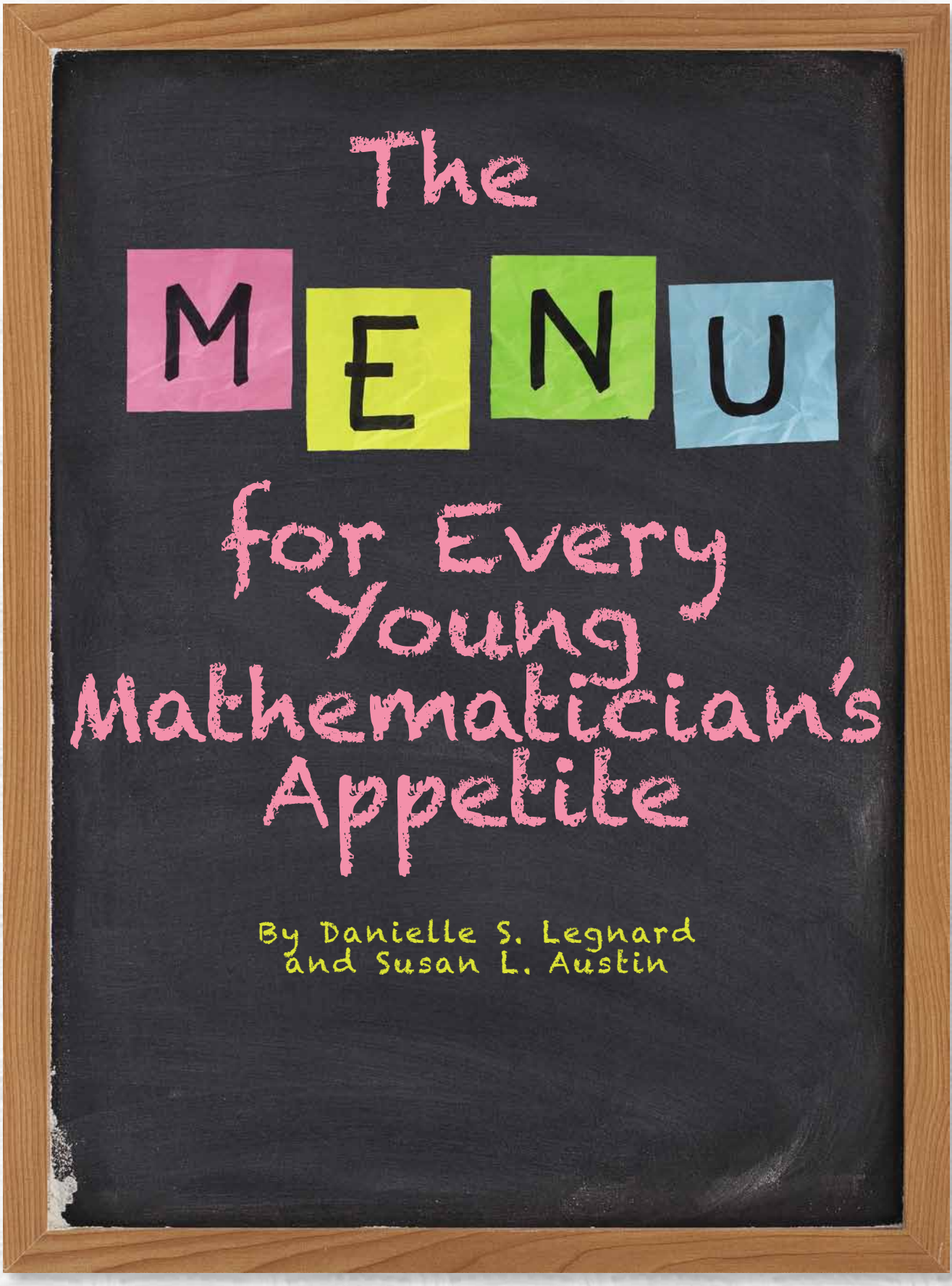
Math Workshop offers differentiated instruction to foster a deep understanding of rich, rigorous mathematics that is attainable by all learners. The inquiry-based model provides a menu of multilevel math tasks, within the daily math block, that focus on similar mathematical content. Math Workshop promotes a culture of engagement and individualization that gives mathematical access to every learner in the classroom community.

In the way that Readers' Workshop and Writers' Workshop build literacy, a mathematicians' workshop will build numeracy. The model presents an excellent opportunity for students to work on meaningful mathematics and understand the big idea behind the lesson or task—making connections to self, text, another problem, or the world. Workshops foster and nurture students' quests for wonder and exploration in a safe risk-taking environment, where the teacher and fellow students collaborate to find deeper meaning behind the mathematics, to algebratize the



The MENU for Every Young Mathematician's Appetite

By Danielle S. Legnard and Susan L. Austin



BLANK DISH: ENE/VEER; KNIFE AND FORK LEAF/VEER; WHITE COTTON FABRIC AS BACKGROUND: INGRIDSI/VEER

The teacher first introduces a skill, a concept, or a strategy. Then she demonstrates, models, and shares her own thinking. Following a focused minilesson, she facilitates a series of small, guided groups.

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arithmetic, to interpret data, and to make generalizations that promote clear understanding of the problem at hand, along with connecting to past and future problems. One teacher admits, "My students are not given enough opportunity to problem solve on their own, and many are resistant when given such an inquiry-based problem or task."

Math Workshop promotes problem-solving routines that build students' stamina and courage. The model also allows a teacher the flexibility to meet with small groups for instruction that can be differentiated on the basis of students' needs, readiness, or interests.

Basic ingredients

In addition to ongoing formative assessments, discussions, and reflections, a Math Workshop includes four key components:

1. A focused minilesson
2. Guided instruction
3. Collaborative practice
4. Independent practice

These four morsels are the pillars of Math Workshop and may occur during the math block, over several days, during the course of a week, or throughout a math topic or concept. The goal is for students to gain a profound understanding of challenging mathematics by working in a variety of flexible groups according to their developmental understanding, strengths and weaknesses, interest, and readiness. In Math Workshop, the teacher's role is to supply differentiated mathematical tasks, problems, and games that build fundamental conceptual understanding and computational fluency; students engage in the mathematics and take ownership of their learning.

An appetizer

The teacher begins a Math Workshop by whetting students' appetites for mathematics with a focused minilesson, during which the teacher demonstrates, models, and shares his or her own thinking with students (Fisher and Frey 2008). Although this initiation is only 5–15 minutes long, it is a crucial occasion for the teacher to introduce the skill, concept, or strategy that he or she is teaching. At this time, the teacher establishes purpose through modeling, metacognitive awareness, and think-alouds (Fisher and Frey 2008). Math Workshop could also be launched with an inquiry-based investigation, experiment, or open-ended task coupled with essential questions and problems.

Just a taste

The first graders in Room 312 are sitting on the carpet, having morning meeting with their teacher. They are counting how many pockets they have on their clothing for a classroom routine called Pocket Day (Economopolous and Wright 1998). First-grade students will count almost anything; they especially love to count their pockets and their missing teeth. To make counting easier, the children focus on finding combinations of ten. During this minilesson, the teacher models on chart paper how to add a string of numbers together using these combinations of ten.

Teacher: Remember our tens. So far, we have 1 ten. Can we find another combination of ten? What goes with six to make ten?

Student A: Four goes with six.
Teacher: How about this five? What goes with this five to make a ten?
Student B: Five goes with five to make ten.
Student C: I found the last group of ten: Eight plus two equals ten.
Teacher: Let's keep track of the combinations of ten we have used. This will help us stay organized [*crossing off the numbers and circling the 4 tens*]. Using our tens makes counting all our pockets easier. How many pockets altogether?
Student D: I see 4 tens and 1 four, so that makes forty-four.
Teacher: How else did we solve this pocket problem?
Student A: I like to bring the numbers together, like a funnel, rather than crossing them off.
Student E: I agree. That is easier for me, too.

The teacher then asks the children about the method they used to count pockets and if that worked for them. She demonstrates her thinking using another number string and models how to make combinations of ten. As the Math Workshop progresses to the next stage, the teacher's role will begin to shift.

Recommendations from the menu

Guided instruction is when the teacher and students do the work together, parallel to one another. During this phase of Math Workshop, responsibility gradually shifts from teacher to student. The teacher's role is to take the learners' lead as they try to apply the skills or strategies that were modeled for them in the minilesson.

Steering youngsters through thinking and learning in a whole-group setting is difficult. Therefore, guided instruction primarily takes place in a small group (Fisher and Frey 2008). During a guided math group, the teacher uses cueing, prompting, scaffolding, and questioning (Fisher and Frey 2008). This critical juncture in Math Workshop allows the teacher to see what a student knows or does not know. Before the teacher can release the student to collaborative or independent practice, the child needs time to absorb the skill or concept that has been taught. While the teacher is meeting with small, guided math groups, the rest of the class engages in authentic, rigorous math tasks, either in collaborative groups or independently. Tasks are often presented on a math menu (see **table 1** on p. 232), which allows students to move through math activities without having to interrupt the small, guided groups.

Guided instruction takes careful planning and consistent routines. The heart of guided instruction is discourse and discussion between teacher and student. These moments are carefully crafted and scaffolded so students can gain clearer understanding and more independence. Such guided instruction helps students bridge understanding with the "tricky parts" (Fisher and Frey 2008).

Tapas

A classroom walkthrough of a morning in room 312 is just a sampling, like *tapas* in the Mediterranean cuisine; but regardless of when you enter the Math Workshop, you will see students working to make sense of the mathematics, building their computational fluency, and problem solving by making connections to their world and what they already know. At this particular moment in room 312—during guided practice—the classroom teacher pulls aside a small group of students who are extending the Pocket Day lesson.

Teacher: How many more to make fifty pockets?
Student A: Well, we had 4 tens and 4 ones.
Teacher: If that is true, how many more to make

Math Workshop promotes problem-solving routines that build students' stamina and courage.

PLATE ON HAND: OLEG KALINAVEER

fifty? How can you use what you know about ten to help you with fifty?
Student B: Well, if I move six cubes over here to make another ten, then I have 1, 2, 3, 4, 5 tens, or fifty.

Such a snapshot in time is a true assessment of a student's thinking about extending tens. While the teacher is working with small groups, guiding them, the rest of the students may choose from a menu that includes collaborative practice and independent practice.

Enjoying the entrée

Students who are not in the small, teacher-facilitated groups are working in pairs, small groups, or independently on a menu of tasks that the teacher carefully designed and differentiated ahead of time. Routines have already been established in the classroom, allowing students to move freely, complete tasks, and progress to math-choice time. Collaborative group work provides an opportunity for students to complete a variety of tasks together. The most effective collaborative group tasks are those that allow students to apply what they have learned from focus lessons and guided instruction (Fisher and Frey 2008). Regardless of the subject matter or content area, students learn more and retain information longer when they work in small groups (Beckman 1990). Teachers should spend a significant amount of time each

TABLE 1

Genuine, rigorous math tasks allow individual students the independence to choose activities and feel like successful young mathematicians.

Math Workshop Menu

Today's special: Room #312 snapshot			
CORE math workshop ingredients	**Flexible groupings**	**Time frame**	**Just-right tasks**
Minilesson	Whole-class instruction	10–15 minutes	Counting Pockets
Workshop tasks • Task A • Task B • Task C	Pairs, collaborative groups, individuals	20–30 minutes	Task A: Tens Go Fish Task B: Turn Over Ten Task C: Quick Images/ Ten Frames
Guided instruction • Group A • Group B • Group C	Flexible small groups meet with the teacher while students work on workshop tasks	10–15 minutes (per group)	Group A: Combinations of Ten Group B: Ten Frames/Getting to the Ten Group C: Using Tens to Get to a Hundred
Discussion and reflection	Whole-class reflection	5–10 minutes	Exit card: *Why are combinations of ten important in our number system?* Journal entry Turn and talk Share
Additional independent practice (homework)	Individualized, tiered, and tailored to meet the needs of learners	10–15 minutes	Tens Go Fish game Using Combinations of Ten Combinations of Ten practice

day with small groups of students. The key in Math Workshop is making sure that the menu of tasks for the rest of the class is engaging and meaningful to students. Fisher and Frey (2008) note that collaborative learning supplies a critical bridge in student learning because it allows novice learners to refine their thinking about new concepts and skills. This is especially true for high-achieving students, who tend to work independently and collaborate less frequently than their peers.

A second helping

During independent learning or practice, students apply what they have learned from the three previous stages. It is also a time when students learn to be self-directed and engaged. Therefore, independent learning tasks must be meaningful, experiential, and relevant (Fisher and Frey 2008). In Math Workshop, the independent learning task or activity can have many forms, such as exit slips, conferring, formative assessments, games, math journal entries, or writing prompts.

Choosing a side dish

Back in Room 312, the teacher has developed and has introduced a menu of math tasks for students to work on as a follow-up to the combinations of tens. She has purposefully selected these activities (see **table 1**) for the rest of her students to work on, with an opportunity for them to make choices during math class. While students are completing their workshop tasks, the teacher takes a series of small groups for guided instruction. These guided groups are based on a variety of needs in the classroom. During this workshop, students are recording their findings and solutions by responding to questions that are designed to be open-ended:

- What did you notice when you played this game?
- Did you figure out more than one way to solve the problem?
- Is there a more efficient way to solve or play?

The questions are carefully crafted to foster reflection and critical thinking. By having the students record their responses, the teacher can zoom in on the children's insights even if she is working with another group of children at the time. This becomes formative assessment and helps the teacher prepare and plan for the next day. Formative assessment is ongoing and takes place in every part of Math Workshop.

Dessert

Bringing children together at the end of Math Workshop is a critical step. Like partaking of the perfect dessert after a fabulous meal, the class uses the time at the end of the lesson to savor essential concepts together. Sometimes the lesson has been accomplished in one class period, and sometimes it has expanded to a few days or a week. No matter the length, each session should end with some closure and reflection. Exit cards and exit slips are an effective tool to gather each child's understanding of the lesson and help the teacher differentiate instruction for the next day. These can be done on note cards or in a journal. The class sharing experience is focused and concise, and like the minilesson introduction, it does not take a lot of time. In just five to ten minutes, much insight can take place. Additional independent practice can be given as homework. Encourage students to play the workshop games at home with the family, which gives parents a great deal of insight into their child's mathematical thinking.

A cheese course

After students have received the minilesson, have had their guided practice, and have completed two or three tasks collaboratively or independently during the day's lesson, it is time to reflect on the entire meal. The teacher gathers them back to the circle to discuss what they have learned. She asks them to think back to the

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Other students work in pairs, in small groups, or independently on tasks from a menu that has been carefully designed and differentiated by the teacher.

beginning of the lesson when they were asked why tens are friendly numbers and how they should best use their friendly tens.

Teacher: What did you learn today that made you a better mathematician?
Student A: Tens Go Fish helps you learn how to make a ten and remember all the ways to make a ten, just like that [*with a snap of her finger*].
Student B: Because tens are important in math, most important.
Student C: I discovered that you could actually break apart one number and add that part on to another number to make a ten, then add on the rest, so you can put it together easily.
Student D: Everything we did today helped us with our tens—even pocket day.

We might frequent fine establishments to try new recipes and might confidently take our guests there because we know we will receive paramount service and attention. So, too, in Readers', Writers', and Math Workshops, we encounter routines and expectations that enable students to be self-directed and responsible for their own learning while ensuring that the teacher has time to work flexibly with every child. Students develop deep comprehension when the teacher provides opportunities for them to experience the interwoven relationship among mathematics, literacy, and thinking. According to *Comprehending Math* (Hyde 2006) and *Principles and Standards* (NCTM 2000), the goal of mathematics teaching is to coach students toward understanding concepts, not merely memorizing facts and procedures. Once teachers realize that rote procedures and memorization do not result in long-term conceptual understanding, maybe they will shift from "traditional" math instruction toward a learning environment where math, language, and thinking are not so much separate entities as they are the necessary elements that true numeracy comprises. After reading *Comprehending Math: Adapting Reading Strategies to Teach Mathematics* (Hyde 2006), a math specialist reflects:

> I've focused on strategies such as questioning and making connections mostly in Language Arts and have now realized they are all just as essential in mathematics, particularly in problem solving. Now it seems so obvious: How can we expect children to be effective problem solvers if they are not able to comprehend the problem itself?

We encounter routines and expectations that enable students to be self-directed and responsible for their own learning.

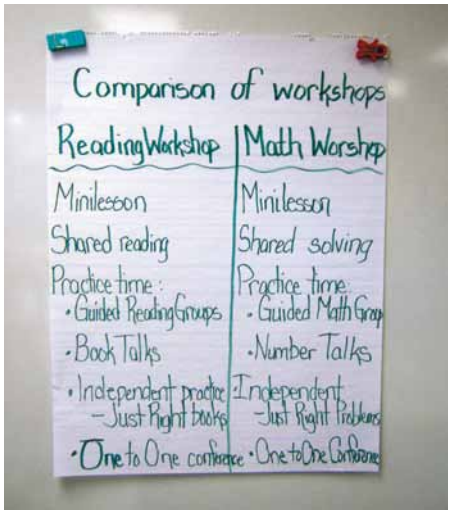
PLATE ON HAND: OLEG KALINAVEER

After dining with the first graders in Room 312 and examining their menu for Math Workshop, it is your turn to plan your own menu and set the table, so to speak, in your classroom. Now, remember that all good chefs (in this case, other elementary school teachers) will make the recipes their own, catering to whomever is joining them for the meal:

> To differentiate instruction, I really need to know my students and how they learn. … Ways I find out how my students learn [are] through observation and by listening to my students answering questions, sometimes responding to a survey. Good teachers also need to know what their students understand by taking time to work with them one-on-one and interview them. I have always known in literacy that students need to read just-right books and answer just-right questions in order to grow as readers, and the same should be done for math. I have found ways to do this in math using the workshop model

FIGURE 1

After a workshop teacher compared literacy and numeracy workshops, she found that the idea of a math workshop "makes perfect sense."



during small-group instruction or during math talks.

The similarities among Readers', Writers', and Mathematicians' Workshops are powerful (see **fig. 1**). A workshop teacher compared literacy and numeracy:

> The idea makes perfect sense. Providing students with a way to make connections will help them develop deeper understanding. The structure of the workshop model in reading, writing, and math can resemble one another. This should be a balanced approach.

The goal of Math Workshop is to work in the *zone of proximal development*, which is the difference between what a learner can do without help and what he or she can do with help (Fisher and Frey 2008). This model allows for discovery and disequilibrium that takes children to the next stage of growth and development and deepens their understanding as readers and writers—and now as mathematicians. Teachers will devour this innovative pedagogy that nourishes their students' minds for a lifetime. Students will be energized by the meal, the menu, and the customized approach to mathematics that meet their individual needs and tastes. Who would not want to feel the satisfaction and success that the workshop model imparts? When one child's teacher asked, "What is a math menu?" his response showed just how eager young mathematicians are to eat this up:

> The Math Workshop menu is like a restaurant menu with lots of stuff on it. It is not plain. If I went to a restaurant, I would not want to see spaghetti, spaghetti, spaghetti, spaghetti, spaghetti on the entire menu. I would want to see spaghetti, chicken fingers, salad, and French fries. That is what our math menu looks like: many different tasks and activities that we need to complete. Some of them we do alone, with a partner, or with our teacher. We also have choices that make us feel like mathematicians.

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This work was supported by the National Science Foundation (NSF). The views expressed in this article are those of the authors and do not necessarily represent those of the NSF.

The authors express special thanks to teachers and students at Stepney Elementary School in Monroe, Connecticut, and West School in New Canaan; Karen Scalzo, a writing coordinator from New Canaan Public Schools; Pamela Khairallah, a workshop teacher previously from Monroe and currently in Weston, Connecticut; and to Math Workshop teachers Eva Kibby, Jessica Koziel, Ashley Conlen, and Karol Fleegal in the Isabelle Farrington School of Education Certificate of Advanced Studies (CAS) in Mathematics at Sacred Heart University in Fairfield, Connecticut.

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